TMT POLISHED MIRROR ASSEMBLY – ROUNDEL POLISHING SPECIFICATION

TMT.OPT.SPE.15.002.CCR07

April 16, 2019

SIGNATURE PAGE

Author Release Note:
See Document Change Record for change requests incorporated at this time.

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DOCUMENT CHANGE RECORD

Revision	Change Request Approval	Release Approval	Date Released
CCR07	CR307 Collection-24217	Docushare Routing #48274	April 16, 2019
CCR06	CR274 Collection-20024 ADMIN Changes	Docushare Routing #35129	June 29, 2018
CCR05	CR237 Collection-15024; Incorporation of Docushare Routings 16600 and 16735 comments. Administrative changes to update ADs/RDs and document standardization as importing to DOORS; Admin change to AD03 post approved proposed changes correcting document version (WAS Version-76499 IS Version-77113).	J. Rogers Routing #17024	7/14/2017
CCR04	Initial CCR release under Configuration Management. CCR04 FINAL - All DRF issues incorporated. Update to AD3, AD7. Signature Cycle comments incorporated. CCR04(DRF01) - 1. Update to AD/RD (links, titles), Cross-references, abbreviations; 2. Update to Appendix A (B. Sutin) 3. Added Appendix B – PSD Calculation Methodology.	See signatures in CCR04	2/19/2016
DRF03	Update in preparation for Initial CCR release: 1) Updated document to current Technical Document Template, TMT.SEN.TMP.14.001.REL01 2) Updated AD and RD section. 3) Corrected minor formatting issues. 4) Renumbered Equations in Appendix A.	-	7/27/2015
DRF02	Updates	-	6/2/2015
DRF01	Initial Draft	-	6/2/2015

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1. INTRODUCTION

The Primary Mirror consists of 492 Segments: 6 instances of each of 82 unique Segment Types. Therefore, 82 unique optical surface configurations are required for Polished Mirror Assemblies in their final form. To allow production of un-mounted polished mirror segments by a Contractor, a specification is required for the mirror segments in their round state. In this configuration, polished mirror segments are referred to as Roundels.

1.1 SCOPE

This specification describes requirements for Roundels or un-mounted polished mirror segments prior to cutting to a hexagonal shape. Roundels are machined to produce an irregular hexagonal shape and add other features, then mounted to a Segment Support Assembly (SSA). The assembly is then a Polished Mirror Assembly (PMA), as depicted in the TMT M1 Polished Mirror Assembly drawing (RD3).

The following Applicable Documents completely define the mechanical and optical characteristics of the 82 types of Roundels:

- TMT M1 Polished Roundel Drawing, (AD3), and
- M1S Segmentation Database, (AD4).

The following specification and drawing are reference documents that provide mechanical and optical characteristics of the Segment Blanks:

- Specification for Primary Mirror Segment Blanks, reference (RD6), and
- TMT M1 Meniscus Segment Blank Drawing, reference (RD7).

The following additional Reference Documents completely define the mechanical and optical characteristics of the 82 types of machined segments which are made from the Roundel:

- TMT M1 Polished Segment drawing, reference, (RD2), and
- TMT M1 Polished Mirror Assembly Drawing, reference, (RD3).

The conventions and instructions for utilizing the M1S Segmentation Database are described in Appendix A (Section 4.1).

1.2 APPLICABLE DOCUMENTS

AD1 DELETED

AD2 DELETED

AD3 TMT M1 Polished Roundel
CAD Drawing No: M1S-001-01001 Rev F
TMT.OPT.DWG.15.008 REL06
https://docushare.tmt.org/docushare/dsweb/Get/Version-77113

AD4 TMT M1S Segmentation Database
TMT.OPT.TEC.07.044 CCR15
https://docushare.tmt.org/docushare/dsweb/Get/Version-61423

AD5 General Specification Governing the Manufacture, Assembly, and Inspection of Optical Components for Fire Control Instruments
MIL-PRF-13830 B
http://quicksearch.dla.mil/Analyse/ImageRedirector.aspx?token=14437.10290

AD6 Dimensioning and Tolerancing
ASME Y14.5 Edition 2009
https://www.asme.org/products/codes-standards/y145-2009-dimensioning-and-tolerancing

AD7 M1 Segment Roundel Specification - Vendor Table
TMT.SEN.TEC.16.009 REL04
https://docushare.tmt.org/docushare/dsweb/Get/Version-98886

1.3 Reference Documents

RD1 TMT M1 Segment Overview and Background Document
TMT.OPT.TEC.09.003
https://docushare.tmt.org/docushare/dsweb/Get/Document-14507

RD2 TMT M1 Polished Segment Drawing
CAD Drawing No: M1S-001-01000
TMT.OPT.DWG.14.005
https://docushare.tmt.org/docushare/dsweb/Get/Document-33140

RD3 TMT M1 Finished Polished Mirror Assembly
CAD Drawing No: M1S-001-11000
TMT.OPT.DWG.07.002
https://docushare.tmt.org/docushare/dsweb/Get/Document-8108

RD4 TMT M1 Intermediate Polished Mirror Assembly
CAD Drawing No: M1S-001-05000
TMT.OPT.DWG.14.004
https://docushare.tmt.org/docushare/dsweb/Get/Document-33139

RD5 Primary Mirror Assembly Integration Overview
TMT.OPT.PRE.10.059
https://docushare.tmt.org/docushare/dsweb/Get/Document-17648

RD6 Specification for Primary Mirror Segment Blanks
TMT.OPT.SPE.07.001
https://docushare.tmt.org/docushare/dsweb/Get/Document-6266

RD7 TMT M1 Meniscus Segment Blank Drawing
CAD Drawing M1S-001-01002, (TMT.OPT.DWG.14.001)
https://docushare.tmt.org/docushare/dsweb/Get/Document-31980

1.4 Abbreviations

AO	Adaptive Optics
CRS	Coordinate Reference System
IBF	Ion Beam Figuring

M1	Primary Mirror
M1S	M1 System
P-V	Peak-to-Valley
PMA	Polished Mirror Assembly
PSA	Primary Segment Assembly
PSD	Power Spectral Density
PSSN	Normalized Point Source Sensitivity
RMS	Root Mean Square
RSS	Root Sum Square
SSA	Segment Support Assembly
TMT	Thirty Meter Telescope
WFE	Wavefront Error

1.5 DEFINITIONS

Acceptance Tests. The term “Acceptance Tests” is defined in Section 1.9.

Back Surface. The Back Surface is the convex surface of the Segment that is on the opposite side from the Optical Surface, also called S2.

Basic Dimension. A Basic Dimension is a dimension that describes the theoretically perfect size, shape, or location of a feature. Geometric Dimensioning and Tolerancing shall be interpreted per ASME Y14.5-2009 (AD6).

Blank. A glass or glass-ceramic substrate from which a hexagonal mirror Segment will be fabricated. The Blanks are described in the *Specification for Primary Mirror Segment Blanks,* reference (RD6).

Blank Supplier. The company that will fabricate the Blanks.

Chip. The term Chip is defined in Section 2.6.6.

Supplier. A company or institution that contracts with the TMT Partner to produce Polished and Mounted PMAs.

Generating. Machining the surfaces of the Blank by fixed-abrasive grinding.

M1 Coordinate Reference System. The M1 Coordinate Reference System (M1CRS) is defined in Section 2.2.

Observatory. The term Observatory refers to the facility on Mauna Kea, Hawaii that will incorporate the Thirty Meter Telescope.

Optical Surface. The Optical Surface is defined in Section 2.6.1, also called S1.

Polished Mirror Assembly. The term Polished Mirror Assembly is defined in Section 1.2.

Primary Segment Assembly. The term Primary Segment Assembly (PSA) is defined in Section 1.2.

PSA Coordinate Reference System. The PSA Coordinate Reference System (PSACRS) is defined in Section 2.2.

Segment. A Segment is one of the hexagonal mirrors that, in combination, form the surface of the TMT primary mirror.

Segment Blank Specification. The Segment Blank Specification is a TMT document listed as reference (RD6).

Subsurface Damage. Cracks in the glass below the surface caused by any process step such as machining or grinding, whether visible or not.

1.6 VERIFICATION METHODS

Included in each major numbered specification listed herein this document is a requirement verification method. These verification methods specify the minimum standards of verification required by TMT to ensure that the individual requirements and specifications are met.

All verification activities are the responsibility of the Contractor; i.e., the Contractor shall be solely responsible for providing any and all test equipment, analyses, inspections, and other means necessary to verify that the specifications and requirements have been met.

Examples of verification methods include:

- **Analysis.** Verification by analysis shall mean that Contractor analytically demonstrates that the design meets the specification. Such analyses may include finite element methods, computation fluid analyses, closed form analyses, etc. All analyses shall be provided to TMT in written report form, in both electronic (e.g., MS Word) and paper copy format.
- **Inspection.** Verification by inspection shall mean that the Contractor visually demonstrates to TMT personnel that the specification has been achieved on the as-built equipment during factory acceptance testing.
- **Acceptance Test or Test.** Verification by test &/or measurement shall mean that Contractor empirically demonstrates that the as-built equipment meets the specification. Testing may be required in the factory during factory acceptance testing and/or at the Delivery Location.

At a minimum, the specification compliance matrix provided by Contractor as part of the Work shall use the verification method(s) listed in each of the requirements sections below.

All analyses, test measurements (with test error analysis) and other verification results shall be provided to TMT in written report form, in both electronic (e.g., MS Word or Excel) and paper copy format. For each test method used for acceptance testing, the Contractor shall perform a test error analysis. All potential errors effecting the measurement shall be listed and their influence on the test results evaluated. The required measurement value shall be adjusted so the Test shall confirm that the specification has been met after taking the test error analysis into account.

2. OVERALL DESCRIPTION

3. SPECIFIC REQUIREMENTS

3.1 GLOBAL PROPERTIES OF PRIMARY MIRROR

The TMT primary mirror is a hyperboloid. The expression for a conic surface of revolution is:

$Z_{M1} (X_{M1},Y_{M1}) = \{R – [R^2 – (K + 1)r^2]^{1/2}\}/(K + 1)$

where K is the conic constant, R is the paraxial radius of curvature, and $r = (X_{M1}^2 + Y_{M1}^2)^{1/2}$. X_{M1}, Y_{M1} and Z_{M1} are in the M1 Coordinate System, as defined below.

3.1.1 Radius of Curvature

[SPE-M1.SEG.ROU-1200] The TMT primary mirror shall have a parent surface paraxial radius of curvature, R, of 60.0 meters. The paraxial radius of curvature shall be considered a Basic Dimension.

Verification: *Test. Polishers shall work with TMT to cross-calibrate their metrology tools such that they are consistent across the supply-chain. Polisher shall provide their reference sphere to TMT for this cross-calibration, and TMT will provide the empirically derived radius offset value to the polisher that shall be used in measuring the polished segments.*

Requirement Origin: *[SPE-M1.SEG.PRE-1100]*

3.1.2 Conic Constant

[SPE-M1.SEG.ROU-1100] The TMT primary mirror shall have a parent surface conic constant, K, of -1.00095348. The conic constant shall be considered a Basic Dimension.

Verification: *Test*

Requirement Origin: *[SPE-M1.SEG.PRE-1100]*

3.2 COORDINATE SYSTEM

The global coordinate system for the TMT primary mirror (M1CRS) is shown in Figure 3-1. Coordinates in this system are designated by the subscript M1, for example: X_{M1}. This M1 Coordinate System is a right-handed system. The Z_{M1}-axis is the optical axis of the telescope, positive towards the sky, and the X_{M1}-axis is parallel to the telescope elevation axis. The positive Y_{M1}-axis points to the sky when the optical axis points to the horizon.

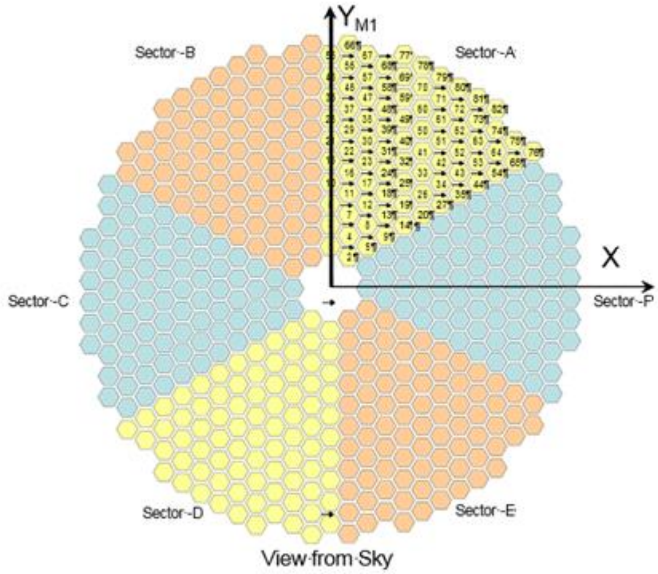

Figure 3-1. The global M1 Coordinate System (M1CRS) for the TMT primary mirror and identification of the six Sectors and 82 Segment Types

The local coordinate system used in this document is based on the orientation of the SSA (PSACRS), and coordinates in this system are designated by the subscript PSA (Primary Segment Assembly), for example: X_{PSA}. The origin of the PSA Coordinate System resides on the Optical Surface at the nominal center of the Segment. The Z_{PSA}-axis is the normal to the Optical Surface at the origin. The X_{PSA}-axis and Y_{PSA}-axis are perpendicular to the Z_{PSA}-axis in a right-handed coordinate system. The position of the origin and the orientation of the X_{PSA} and Y_{PSA} -axes have been chosen to provide the best orientation of the Segment with respect to the support system for each Segment Type. The Segments and PSA coordinate system rotate by 60 degrees when segments are moved from one sector to a neighboring sector, etc.). As such, the projection of the X_{PSA} and Y_{PSA} -axes will not, in most cases, coincide with the Segment vertices, and they will not be parallel to the X_{M1} and Y_{M1} axes. The mechanical dimensions of the Segment are defined with respect to the PSA Coordinate System. More details on the definition of the PSA Coordinate System are provided in Appendix A (Section 4.1).

3.3 THEORETICAL OPTICAL SURFACE SHAPE

[SPE-M1.SEG.ROU-2026] Roundels shall have individual theoretical surface shapes described by Zernike terms as defined in (AD4). Zernike terms shall be treated as Basic Dimensions and centered on the mechanical center of the Roundel without tolerance allowance.

Verification: *Inspection*

Requirement Origin: *[SPE-M1.SEG.PRE-2026]*

3.4 POLISHED ROUNDEL REQUIREMENTS

[SPE-M1.SEG.ROU-1420] Polished Roundels shall conform to all of the requirements of (AD3).

Discussion: Segment Blanks used to fabricate Polished Roundels for the PMAs shall conform to all of the requirements of (RD6) and (RD7).

Verification: *Inspection*

Requirement Origin: *[SPE-M1.SEG.PRE-1430]*

3.5 CLEAR APERTURE

[SPE-M1.SEG.ROU-1900] The "clear aperture" over which the optical figure error (Sections 2.6.1and 2.6.2) applies is the entire concave surface of the Segment that will remain after proposed hexagonal trimming, extending to the edge of the protective bevels on all sides.

Verification: *Inspection*

Requirement Origin: *[SPE-M1.SEG.PRE-1900]*

3.6 OPTICAL SURFACE REQUIREMENTS

3.6.1 Optical Surface Figure Error (low spatial frequency)

[SPE-M1.SEG.ROU-1920] Requirements specified herein are for Roundels prior to machining to a hexagonal shape and Ion Beam Figuring (IBF). Optical surface figure errors relative to the theoretical surface shape for each Roundel shall be less than 1.73 microns Peak-to-Valley (P-V).

Rationale: *Combining the 1.73 micron P-V maximum error with a 1.0 micron P-V error due to change in shape, as a result of hexagonal machining, results in the 2.0 micron P-V maximum figure error required of the Polished Segment. The TMT Project is assuming the risk associated with low order deformation due to hexagonal machining that is greater than 1.0 micron P-V.*

Verification: *Test*

Requirement Origin: *[SPE-M1.SEG.PRE-1920]*

3.6.2 Optical Surface Figure Error (high spatial frequency)

[SPE-M1.SEG.ROU-1410] The estimated figure error metrics of the Optical Surface clear aperture of the roundel after machining, IBF correction and warping harness adjustment shall meet the following requirements as calculated by (AD7):

- PSSN shall be greater than 0.96 (as calculated by cell Q8 of (AD7)).
- Adaptive Optics WFE shall be less than $15nm_{RMS}$ (as calculated by cell V8 of (AD7)).
- Maximum stroke of any warping harness actuator shall be less than 20%, to correct low frequency errors (as calculated by cell AA8 of (AD7)).

Discussion: (AD7) is applied the measured figure error of the roundel to predict IBF and warping harness correction to the machined, mounted segment after IBF. (AD7) then calculates the effect of these residual errors on PSSN, post AO WFE and warping harness stroke. The results can then be compared to the above requirements. Table 3-1 below describes the steps necessary to calculate the inputs to (AD7).

Table 3-1. Process for calculating inputs to Roundel and Intermediate Segment Vendor Table (AD7)

	Process	**Output**
1	Define a circular region on the radius of curvature and conic compensated mirror under test with a diameter of 1:44 m. Define the Zernike co-ordinates as in Figure 3-2.	
2	Calculate Zernike coefficients up to 15 Noll modes for the region based on measurements of the optical surface within the Clear Aperture. Define these values as measured LOZ and denote them as $Z_{M,i}$ where i = 1, 2, ::15. An additional 1 micron of springing shall be assumed as a focus change and combined with the measured focus error.	measured LOZ
3	Calculate the mid-frequency aberration rmsSRF and denote it as $rmsSRF_M$. The mid frequency aberration is defined as "LOZ re-moved residual surface up to 20 cycles/m (or 50 mm period)".	measured $rmsSRF_M$
4	Calculate the high frequency aberration rmsSRF and denote it as $rmsSRF_H$. The high frequency aberration is defined as "LOZ removed residual surface between 20 cycles/m (or 50 mm period) and 1250 cycles/m (or 0.8 mm period)".	measured $rmsSRF_H$
5	Calculate the surface roughness rmsSRF and denote it as $rmsSRF_{SR}$. Surface roughness is defined as "surface aberration whose spatial frequency is higher than 1250 cycles/m (or 0.8 mm period)".	measured $rmsSRF_{SR}$
6	The measured surface quality metrics are input under Surface Error Values (Inputs) of (AD7).	
7	Evaluate measurement uncertainty of each error term and input them under "Systematic Measurement Uncertainty" (column D) of (AD7).	uncertainty of the metrics
8	Total errors shall be computed as RSS of Measured Surface Error and Measurement Uncertainty. They shall be located under "Total Error, RSS" (column E) of (AD7).	LOZ, $rmsSRF_M$, $rmsSRF_H$, and $rmsSRF_{SR}$

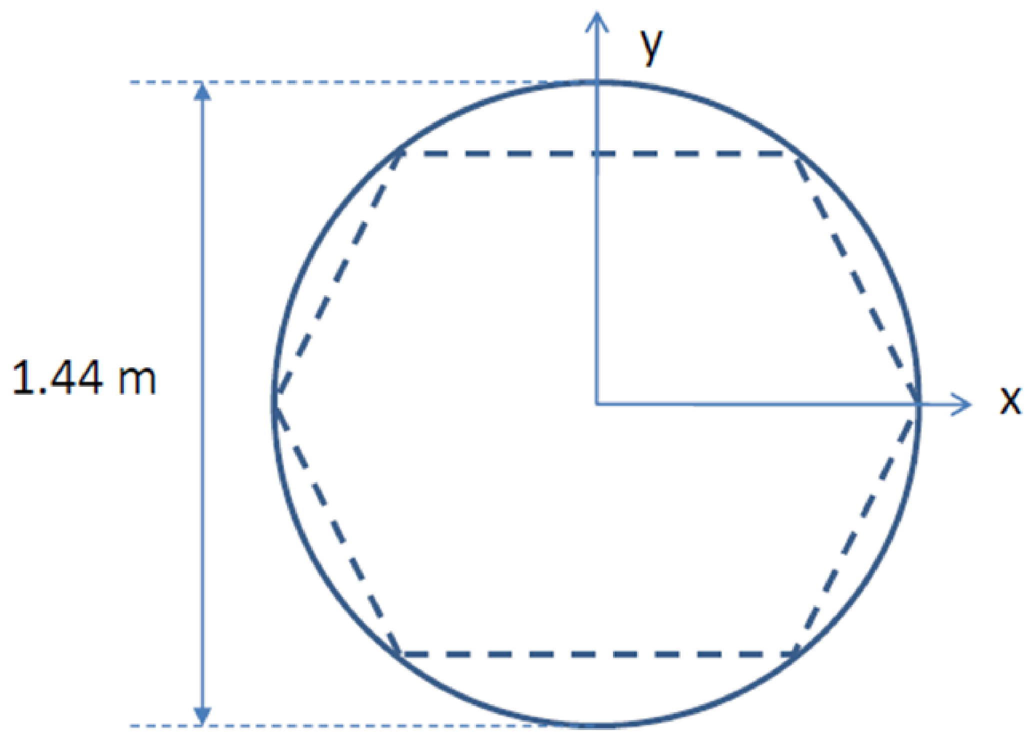

Figure 3-2 - Coordinate system definition for segment Zernikes. Zernikes are defined for the circular aperture of the mirror under test with diameter of 1.44m.

3.6.3 Polished Surface Roughness

[SPE-M1.SEG.ROU-1930] The maximum surface roughness of both front (S1) and rear (S2) polished surfaces shall be 20 Angstroms RMS. The roughness measurement shall capture all features with a spatial period between 20 and 800 microns.

Verification: *Test, results shall be averaged over 20 random locations per surface.*

Requirement Origin: *[SPE-M1.SEG.PRE-1940]*

3.6.4 Polished Surface Cosmetics

[SPE-M1.SEG.ROU-1940] The scratch-dig requirement for the front (S1) polished surface is 60-40. The scratch-dig requirement for the rear surface (S2) is 80-50.

Discussion: The first number is the maximum width of a scratch in microns and the second number is the maximum diameter of digs in units of 0.01 mm, U.S. Military Surface Quality Specification, MIL-PRF-13830B (AD5).

Verification: *Inspection*

Requirement Origin: *[SPE-M1.SEG.PRE-1960]*

3.6.5 Cracks

[SPE-M1.SEG.ROU-2110] No visible cracks shall be allowed in the Roundel. If a crack develops in a Roundel surface, the crack shall be ground-out, leaving a depression that is approximately spherical. The depth of any such spherical depression shall be less than half the diameter of the sphere. A ground out spherical depression shall be considered to be a Chip as defined in this specification.

Verification: *Inspection*

Requirement Origin: *[SPE-M1.SEG.PRE-1990]*

3.6.6 Chips

[SPE-M1.SEG.ROU-2115] No Chip shall exceed 10 mm in mean diameter after grinding. No more than three Chips are allowed on any Roundel. No more than one Chip is allowed on the Optical Surface, and no more than one Chip is allowed on the Back Surface. No Chip is allowed to be within 10mm of any bonded support location or within 10mm from an edge sensor mounting location.

Defijnition: *A Chip is defined as a hollow depression in a surface of the Roundel, usually formed where a flake has broken out of the Blank. All surfaces of a Chip must be ground out to remove sharp edges and cracks.*

Verification: *Inspection*

Requirement Origin: *[SPE-M1.SEG.PRE-2000]*

3.7 Marking Requirements

3.7.1 Serialization Marks

[SPE-M1.SEG.ROU-4010] A unique part number shall be permanently applied to each Roundel per (AD3).

Verification: *Inspection*

Requirement Origin: *Engineering drawing*

3.7.2 Alignment Marks

[SPE-M1.SEG.ROU-4020] An alignment fiducial indicating the orientation of the optical surface shape shall be permanently applied to each Roundel per (AD3).

Verification: *Inspection*

Requirement Origin: *Engineering drawing*

4. TESTING REQUIREMENTS

4.1 SPATIAL PERIOD COVERAGE

[SPE-M1.SEG.ROU-2010] Testing shall be performed in order to determine the surface figure of the segment over all spatial periods equal to or greater than 0.8mm. In order to fully characterize the high frequency surface content and establish a PSD, continuum, it may be necessary to engage complementary and spatially overlapping surface data from more than one test method. Measurement process shall be submitted to TMT for prior approval.

Discussion: Because the control of the radius of curvature of the Segment is critical, no amount of focus term may be subtracted from the test data. In addition, no Zernikes (other than piston, tip and tilt) are allowed to be removed from the test data. The full Zernike content is used in evaluating the Peak to Valley surface specification. Validated test-set-specific systematic error contribution removal may be permissible pending TMT written prior approval.

Verification: *Test or Analysis*

Requirement Origin: *[SPE-M1.SEG.PRE-2010]*

4.2 OPTICAL SURFACE TESTING CONFIGURATION

4.2.1 Testing Support

[SPE-M1.SEG.ROU-2020] The Acceptance Test of each Roundel shall be performed with the Roundel supported in a manner that minimizes gravity induced deformations. Uncertainty in figure due to the support configuration shall be incorporated into optical figure testing error analysis and submitted to TMT for prior approval.

Verification: *Inspection*

Requirement Origin: *[SPE-M1.SEG.PRE-2020]*

4.2.2 Testing Orientation

[SPE-M1.SEG.ROU-2022] Acceptance Testing shall be performed with the Optical Surface facing upwards, i.e. with the Z_{PSA}-axis vertical.

Verification: *Inspection*

Requirement Origin: *[SPE-M1.SEG.PRE-2022]*

4.2.3 Testing Temperature

[SPE-M1.SEG.ROU-2028] The Acceptance Test shall be performed with the Roundel and its support system at a uniform temperature to within 1°C between 18 and 25°C.

Verification *Inspection*

Requirement Origin: *[SPE-M1.SEG.PRE-2028]*

4.3 OPTICAL SURFACE TESTING ACCURACY

[SPE-M1.SEG.ROU-2034] All tests and measurements used to verify compliance with the requirements of this specification shall be of sufficient accuracy to ensure the requirements have been met with a 1-sigma probability. This means that the measured values shall be sufficiently within the allowable range that, when measurement error is included, there is a 1-sigma probability that the parameter being measured is in compliance with the requirement.

Verification: *Analysis*

Requirement Origin: *[SPE-M1.SEG.PRE-2034], [SPE-M1.SEG.PRE-2038]*

4.4 DEMONSTRATION OF SUBSURFACE DAMAGE REMOVAL

[SPE-M1.SEG.ROU-1935] Processes used to remove subsurface damage (e.g. polishing or etching) shall be qualified. Qualification shall demonstrate that the processes used in the production of the Roundels routinely produce surfaces that are free of Subsurface Damage, using the TMT Blank material.

Rationale: The Optical Surface must be fully polished and free of Subsurface Damage that would increase the surface roughness of the polished surface after 2 microns of material removal using IBF.

Verification: Test by Ion Figuring representative coupons; Process shall be submitted to TMT for prior approval.

Requirement Origin*: [SPE-M1.SEG.PRE-1940]*

5. APPENDIX

5.1 APPENDIX A. DESCRIPTION AND INSTRUCTIONS FOR USING M1S SEGMENTATION DATABASE (AD4)

5.1.1 Primary Mirror Coordinate Systems and Associated Notation

Two sets of coordinate axis systems are used to describe the primary mirror:

- a global system, designated as the *M1 Coordinate System*, and denoted by the subscript _M1;
- a series of local systems designated as *PSA Coordinate Systems*, and denoted by the subscript _PSA. There are as many PSA systems as there are segments.

All axis systems use the normal conventions for right hand, orthogonal Cartesian systems; in particular, positive rotations are always in the conventional, right hand direction relative to the coordinate axes.

Each one of the 492 segments in the array has its own, unique, local coordinate system $(xyz)_i$. Because of the 6-fold symmetry of the array, however, local systems (and segment geometry) need only be defined once for each of the 82 segment Types. When going from one sector to another, the segments, support systems, and local coordinate systems, all rotate together by multiples of 60° about the global Z axis.

The local system is defined in such a way that the location of the segment support system (axial support points, etc.) relative to the local axis system is identical for all segments.

The origins of the coordinate systems are denoted as $O_{PSA,i}$ and O_{M1}. Unit vectors along the positive direction of the coordinate axes are denoted $\vec{1}_{X,M1}$, $\vec{1}_{Y,M1}$, and $\vec{1}_{Z,M1}$ for the global system, and $\vec{1}_{X,PSAi}$, $\vec{1}_{Y,PSAi}$, and $\vec{1}_{Z,PSAi}$ for the local system attached to segment #*i*.

5.1.2 Segment and Vertex Numbering

Segments within sector A are numbered from 1 to 82 as shown in Figure 3-1 of the main document. The same numbers are used to distinguish segment Types. Within each segment, vertices are numbered counter-clockwise from 1 to 6, starting with the vertex closest to the positive local X_{PSA} axis, as shown on the drawings, (RD2) and (RD3).

5.1.3 Detailed Definition of the Local Segment Reference System (XYZ_{PSA})

The local PSA axis system results from a series of mathematical operations that are designed and adjusted to maximize the expected performance of the segment support system, by minimizing the deviation between actual segment outlines and a nominal, regular hexagon used in the design of the support system.

Because of this, there is no simple definition of the local frames $(XYZ)_{PSAi}$.

Section 2 of the segmentation database, (AD4) contains definitions of the local PSA coordinate systems (in sector A), expressed as follows:

- the coordinates of the center of the segment (O_{PSA}), expressed in the M1 system
- the components, expressed in the M1 system, of unit vectors $\vec{1}_{X,PSA}$, $\vec{1}_{Y,PSA}$, $\vec{1}_{Z,PSA}$ along the coordinate axes X_{PSA}, Y_{PSA}, Z_{PSA}.

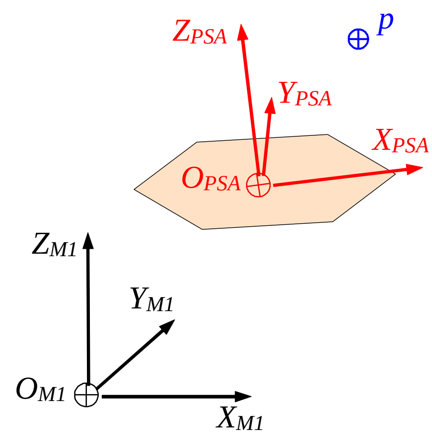

Figure 5-1. Coordinate transformation between the M1 system and a PSA system.

Given the coordinates of any point p, expressed in the local PSA axis system (p_{PSA}) (Figure 4-1), the coordinates of the same point, expressed in the M1 system (p_{M1}) are given by

$$p_{M1} = O_{PSA_{M1}} + \left[\vec{1}_{X,PSA_{M1}} \quad \vec{1}_{Y,PSA_{M1}} \quad \vec{1}_{Z,PSA_{M1}}\right] p_{PSA} = O_{PSA_{M1}} + R_{M1}^{PSA} p_{PSA}, \quad (1)$$

where $O_{PSA_{M1}}$ is a column vector (3×1) containing the coordinate of the origin of the PSA system, expressed in the M1 system, and $R_{M1}^{PSA} = \left[\vec{1}_{X,PSA_{M1}} \quad \vec{1}_{Y,PSA_{M1}} \quad \vec{1}_{Z,PSA_{M1}}\right]$ is the rotation matrix (3×3) from the M1 system to the PSA system, formed as the juxtaposition of the three column vectors ($\vec{1}_{X,PSA_{M1}}$, $\vec{1}_{Y,PSA_{M1}}$, and $\vec{1}_{Z,PSA_{M1}}$) containing the components of the unit vectors of the PSA system, expressed in the M1 system.

Note that since R_{M1}^{PSA} is ortho-normal, the inverse transformations is

$$p_{PSA} = \left(R_{M1}^{PSA}\right)^T \left(p_{M1} - O_{PSA_{M1}}\right),$$

where the subscript ^{T} denotes the transpose of the rotation matrix.

5.2 Appendix B. Definition of one-dimensional PSD of two-dimensional data

The two-dimensional PSD of a two-dimensional surface z(x,y) is defined such that the integral

(1) $\iint psd(f_x,f_y)\, df_x\, df_y$

over some region in the frequency plane is the square of the RMS of all Fourier components of z(x,y) with frequencies in that region. Each spatial frequency pair (f_x,f_y) represents a sine wave of the form

(2) $\exp(2\pi i(x\, f_x + y\, f_y))$.

If we define f and θ by $f_x = f \cos\theta$ and $f_y = f \sin\theta$, then this can be written as

(3) $\exp(2\pi i f(x \cos\theta + y \sin\theta))$,

which represents a simple sine wave with spatial frequency f at orientation angle θ in the xy plane. The actual spatial frequency represented by the frequency pair (f_x,f_y) is therefore $f=\sqrt{(f_x^2+f_y^2)}$. This means that points on a circle of radius f centered at (0,0) in the frequency plane all represent the same spatial frequency f.

Using polar coordinates in the frequency plane, the RMS for all spatial frequencies between 0 and f is given by

(4) $RMS(f)^2 = \int_0^f \int_0^{2\pi} PSD(f \cos\theta, f \sin\theta)\, f\, d\theta\, df$.

The one-dimensional PSD function should be defined from this as $PSD(f) = (d/df)\, RMS(f)^2$, or

(5) $PSD(f) = f \int_0^{2\pi} PSD(f \cos\theta, f \sin\theta)\, d\theta$.

With this definition of PSD(f), the RMS of all surface components (at all orientation angles) in the frequency band from f1 to f2 is given by the area under the curve, that is,

(6) $RMS(f1,f2)^2 = \int_{f1}^{f2} PSD(f)\, df$.

In practice, we calculate the integrals in (5) and (6) by the trapezoidal rule, using linear or bilinear interpolation to evaluate functions between grid points.